

# INQUIRIES ON CARNAP'S AUFBAU [I]

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## 1. INTRODUCTION

Nowadays, Carnap's AUFBAU is getting new significances being considered from the viewpoint of a general theory of representation. We intend in the present paper, to make clear the possibility of this interpretation. Therefore, it is necessary to think of another way of approaching the AUFBAU,<sup>1</sup> which tries to definitivate the theory of constitution, without going beyond the boundaries of the system, but reconsidering each step of it. In 2., a general review of Carnap's construction is given, then in 3 we outline some distinctions, crucial for the thorough understanding of the AUFBAU. In 4 we describe the constitutional reality and its four levels, focusing mainly on the first two of them. In 5 we draw the constitutional ideality, and in 6 we reveal the numerical representational foundations of the basic (abstract) level, evincing by the introduction of the concepts of *interval* and *resolution power*, a modality to formalize the quasianalysis. To facilitate the apprehension of our approaching to the carnapien system we offer, in 7, a pictorial explanation of it.

## 2. GENERAL REVIEW

The difference between what is only a clue in Russell's '*Our knowledge of the External World*', and its pursuance in Carnap's AUFBAU is equivalent to the one between 'the small step for man' and 'the huge leap for mankind'. There were many attempts to construct a world-system based on the sense data (Mach, Avenarius, Jacoby, Driesch). At the time, the second half of the last century, Boole, Frege and, later on, Russell and Whitehead developed a new pattern of thinking: the mathematical logic. The strength and coherence of Carnap's system will be reached by fusing these two major

<sup>1</sup> We overlook Goodman's undertaking of explaining and refuting the quasianalysis caused by some unfavorable circumstances, a way of understanding the AUFBAU appropriated by most of the authors.

biases. An old philosophical claim to put into a system all that can be said of the world is reconsidered now within the compass of a new conception about object, function, concept and relation. If, in 1906, Poincaré said that ‘only the relations between the sensations have an objective value’, Carnap pretends to advance from relations to the structure of relations (§16). The structural approach to the system’s material basis presuppose an *epoché* of the sense data’s psychological content. Consequently, the carnapien procedure does not belong to the *theory of knowledge*, but to the *theory of the logical constitution of knowledge*. In other words, it concerns the ‘*rationale Nachkonstruktion*’ of the real cognitive process, which in every day life is intuitive, as the philosopher emphasizes many times.

### 2.1 This distinction between the theory of knowledge and the theory constitution will be made clear by using different languages, marked with two kinds of symbols: p...p and c...c, respectively

Focusing on Carnap’s system, there is to be mentioned that it was conceived and achieved within the scope of contemporary scientific lights. That means it could be improved by new modern acuirements of sciences, but, Carnap sustained, the logical structure of the system remains the same. Nevertheless, in his preface for the AUFBAU’s second edition, the author writes he would have preferred another beginning for the system: more basic relations, and the replacement of elementary experiences by ‘something similar to Mach’s elements, i.e. concrete sense data, as for example, a red of a certain type in a certain place at a given time.’ Another mention regards the basis of the system: it is autopsychical (*eigenpsychische*).<sup>2</sup> Materialiter, it consists in sense data conceived as stream of experience, unitary and indivisible, which is methodologically sectioned (i.e. through the method of rational reconstruction) into temporally ordered elementary experiences. Formaliter, it is the relation Rs, the recollection of similarity between elementary experiences.

The elementary experiences built by the relation of part similarity obtained out of Rs, a class called similarity circle. This is the first step of quasianalysis, a procedure invented by Carnap to handle with elementary experiences as ‘unanalysible units.’ The common area, resulted from overlapping more similarity circles represents the quality class that is assigned as a quasic constituent part to each of the participating elementary experiences. After the quality classes the sense classes follow (firstly, the visual sense

<sup>2</sup> In our opinion it is wrong and misleading to translate “*eigenpsychische*” by “auto-psychological” as George’s version of the AUFBAU does. One must distinguish between these two planes: the psychological and the psychical. The first belongs to the theory of knowledge, the second to the theory of constitution (see 2.1)

class), consisting of qualities belonging to the same sensitive modalities (§85). The next step is the construction of temporal order (§87) on the basis of Rs. Then, each sense class has a different dimension number. The auditory sense has 2, the visual one, 5. Any visual quality (colour) is determined by hue, saturation, brightness and by local sign (*Lokalzeichen*). Two visual qualities are place identical if they agree in local sign, and they are colour identical if they agree in hue, saturation and brightness. Furthermore, the places of the visual field could be defined as abstraction classes of the relation place identity, i.e. as the largest possible classes of place identical qualities, if place identity had already been defined (§88). But, in §118, the relation place identity is derived from the class place. *Circulus vitiosus?* Out of various groups of overlapped similarity circles one obtains more quality classes. By the relation of quality identity (in particular, colour identity) there results abstraction classes of quality classes (within the visual field, colours). Then, in §89, the spatial order of the visual field is defined as the visual field places class, built by the relation of place proximity. The colour order, as quality class, is constituted by the relation of colour proximity. The psychical sensations (see note 2) are brought in by the temporal individualisation of quality classes (§93). Here ends the construction of the psychical level of the constitution system.

### 3. DIFFERENT LEVELS OF READING

To provide a correct interpretation of Carnap’s theory, one must distinguish, at first, between the psychological and the constitutional planes (see 2.1. and note 2). Another crucial distinction to be made concerns the two sorts of accounting the theory. The one on the real human subject level, marked with the c...c symbols, the other on the fictitious subject level, denoted by us with the cA...cA symbols.

Carnap introduces the fictional plane out of pure didactical reasons, for ‘the constitution system itself has nothing to do with these fictions’ (§99). Consequently, one can speak of distinct language types. On the one hand, proper to the theory of knowledge, the psychological language, on the other hand, belonging to the constitutional theory, the fictional language for the ideal level, and the logistic, the word and the realistic languages, for the real level.

Accordingly, one can speak of three kinds of subjects: a psychological one (his knowledge is provided by psychological sensations), a real constitutional one, which deals with elementary experience, and the ideal (fictitious) constitutional subject, who operates with number-pairs list built on the basis of cA Rs cA, numbers that denote elementary experiences.

Nevertheless, the subject *cA* does not have access to the stream of elementary experiences (as the subject *c* does), but to the number list only. Similarly, there are three types of objects: the psychological one representing an amount of psychological sensation that can be described and characterized; the real constitutional object consisting in a list of elementary experiences pairs, on the basis of *cRsc*, and the last one, the ideal constitutional object being an inventory list (*Bestandlist*) as a number list to which an object description is added (§102).

There are, also, different meanings of *Rs*. On the *p*-plane one can define *Rs* as it follows:  $p \text{ hRsg } p = p \text{ h}$  is earlier than and similar with  $g \text{ p}$ , where *h*, *g* are psychological sensations. On the *c* plane, Carnap defines *Rs* in psychological terms: '*c* *xRsy* *c* means *p* *x* and *y* are elementary experiences which are recognized as part similar through the comparison of a memory image of *x* with *y* *p*' (§78, §108). In other words, it would be  $c \text{ xRsy } c = p \text{ xRsy } p$ . This definition proves to be twice inconsequent: on the one hand, the psychological plane is intermingled with the constitutional one. On the other hand, Carnap uses terms like  $p \text{ xp}$ ,  $p \text{ yp}$ , i.e. elementary experiences on the *p*-plane. Or, it is improper to speak of elementary experiences here, since elementary experiences are methodological sections of the stream of experiences on the real constitutional plane, as it would be on the ideal one. A rigorous and consequent definition of *cRsc* means *c* *x* and *y* are elementary experiences in which similar quality spots occur *c*. On the *cA* plane, *cA* *Rs* *cA* means *cA* the double occurrence of a number *cA*. The things become clearer by making the meaning of the stream of experience explicit. *p* Stream of experience *p* would be a continuous amalgam of psychological sensations. *c* Stream *c* is a list of elementary experiences. *cA* Stream *cA* consists in the basis list of number pairs.

#### 4. THE CONSTITUTIONAL REALITY

On the real constitutional plane we are to distinguish between four levels:

a) the methodological-abstract level, where one can speak of elementary experiences, quality spots and local signs, and by relating them, of methodological temporal, qualitative and spatial orders. Carnap does not mention the last one, the first two being comprised in *Rs*. We argue below that the methodological spatial order is compulsory and, more, the other two are sustained by it.

b) the psychic-preliminary level, which includes visual places, qualities (*Qualitätskörper*) and psychical sensations. Accordingly, there are preliminary temporal, qualitative and spatial orders.

c) the physical-complete level incorporates physical objects completely determined spatially, temporally and qualitatively.

d) the level of physics: magnitudes and orders of physics theories. Furthermore, we explain the meanings of these three basic concepts: time, quality, space.

*Time.* The methodological temporal order is given by methodologically sectioning the stream of experience (§67). The preliminary temporal order is obtained through the relation *Rs* between elementary experiences; it is an interrupted (*lückenlose*) order (§87, §120) that is to be supplemented, on the physical level, by regularities of physical processes. Finally, the temporal order of physics is theoretized by physics conceptions.

*Quality.* On the first level, there are quality spots, i.e. abstract representations of quality (§25: Carnap particularized quality as colour, and speaks about colour spots). The quality spot is without spatial temporal and qualitative determinations, which concern the content. On the second level, there are quality classes, that is *Qualitätskörper* (colour classes, to wit, *Farbkörper*) (§77). By colour solid (*Farbkörper*) and tone scale (*Tonreihe*) one must not understand a three-dimensional volume object, and, respectively, a surface, but a three dimensionality given by hue, saturation, brightness and a bidimensionality delivered through pitch and loudness. Carnap speaks, also, (§72), of spheres of colour (*Farbkugeln*) meaning 'the largest possible parts of the colour solid, which contain nothing but colours that are similar to one another, are spheres which partially overlap each other, and whose diameter is the arbitrarily fixed (*festgesetzt*) maximal distance of similarity'. Colour solids and spheres contain colours, while colour classes and, respectively, colour similarity circle comprise colour things.

**4.1 We emphasize as being the *keynote* of the whole system, the determining of similarity between colours through the stipulation (*Festsetzung*) of a sphere diameter within which the colours are in agreement, that is to say, by fixing a *similarity interval***

On the third level, quality regards physical objects with complete qualitative determinations, i.e. coloured physical objects. On the fourth level, quality translates a magnitude (§10: colour corresponds to a frequency of the oscillating light).

*Space.* The most embarrassing for the *Konstitutionssystem* is the concept of space. On the abstract level, Carnap refers to the mathematical abstract space (§25). In our opinion this space is represented through *local signs*, which is one of the fundamental concepts of the AUFBAU, insufficiently clarified by the author. The deficient apprehension of the local sign brings Carnap on the way to the impossibility of defining the visual field place (§88, §118), on the preliminary level. The only possible definition of the visual field place, we believe, is based on the local sign (similar with the definition of the psychical sensations on the basis of elementary experiences, and of the

quality classes on the basis of quality spot). Then, one could ask about the definition of the local sign. The visual field bidimensionality (meant by Carnap not as phenomenal surface) rests on the abstract bidimensionality of the local sign. We believe the latter can be expressed by Cartesian coordinates. Carnap writes in §25, §125 about spatial configurations (e.g. a sphere) deprived of any determinations, differing from the abstract mathematical space through not being expressions of an abstract, nonspatial geometry, but concrete and spatial. These configurations are distinct from the physical objects, too, since they do not have spatial, temporal, colour and weight determinations.

On the physical level, there is a complete threedimensional space, built on the basis of the bidimensionality of preliminary space (cf. also "Dreidimensionalität...").

On the level of physics, there is taken into account the non-Euclidean space of the physics theories.

These three concepts of time, quality and space fall under the more general concept of representation. So, we will speak of four kinds of representations: abstract, concrete, complete and of physics.

### 5. THE CONSTITUTIONAL IDEALITY

In this section we depict the constitutional ideal plane. The cA-subject starts from a first number list that is provided to him, as to the c-subject is given the stream of experience. On the c-plane the stream is structured by elementary experiences, spots and local signs (i.e. through numerical representations); on the cA-plane cA the stream cA is formed by a prime list of number pairs. Based on this, the cA subject constructs other lists corresponding to the similarity circles, through the occurrence of the same number. On this cA-plane one works with the strong relation of identity, while on the c-plane with the weak relation of similarity. Each cA-constituted object represents a number list whose elements are other lists of numbers, and so on, until the first list.

### 6. THE A PRIORI INTERVALS

The similarity circles are a maximal class of elementary experiences, formed on the basis of part similarity relation. But, *what is similarity?* In §27, Carnap constitutionally defined the Ps relation in psychological language, this being an inconsequence. The correct definition would be: two elementary experiences x and y are called part similar if an experience quasiconstituent (i.e. a quality spot) A of x and an experience quasiconstituent B of y agree in

then local sign. Out of this agreement result maximal classes of quality spots (§111). *What does this agreement mean?* To be in agreement is to be ordered by 'that which orders' (local sign) As Carnap emphasizes in §91, the difference between the local sign and the quality spot is that only the first is a *principium individuationis*, but the second is not. Hence, the role of the local sign is to order, and that of the spots, to be ordered:

*What does to order the quality spots mean?* Colour similarity (namely, on the preliminary level) is defined (see 4.1) by stipulating an interval within which the colours are *considered* similar.

Analogously, similarity between quality spots rests on the local sign agreement. Two local signs agree if the values of Cartesian coordinates are comprised in these intervals, respectively. In other words, two elementary experiences x and y become related by Ps if they contain two spots A and B, respectively, with their local signs [A (a<sub>1</sub>, b<sub>1</sub>), B(a<sub>2</sub>, b<sub>2</sub>), where a and b are numbers] in agreement. The two local signs agree if they belong to the marked area (see Fig. 1), that is if {(a<sub>1</sub> ∈ [m, n] and a<sub>2</sub> ∈ [m, n]) and (b<sub>1</sub> ∈ [q, r] and b<sub>2</sub> ∈ [q, r])}, where [m, n], [p, q] are *a priori* stipulated intervals<sup>3</sup>. *A priori* stipulated are, also, the logical rules of constitution (§103), which are used at each step of the system, as these intervals are, too. The measure of this interval becomes a major characteristic of the constitutional system.

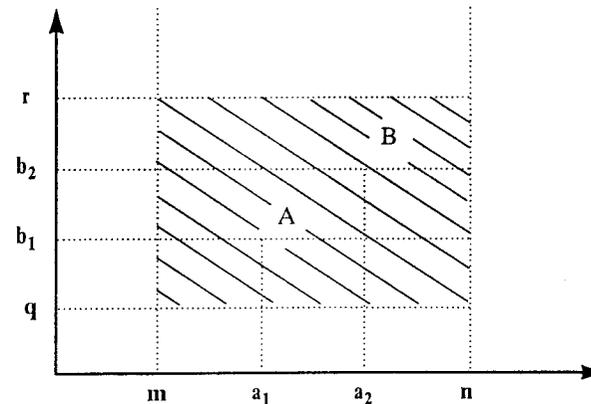


Fig. 1 Two elementary experiences graphically superposed

<sup>3</sup> We have to say that the local sign structure of an elementary experience, i.e. the abstract spatial order, is not to be understood as a lexicographical ordering of a plane, for, in this case, a function *f* from real numbers (Cartesian coordinate values) to marked areas (determined by intervals) would have to be a one-one function describing an isomorphism between the model of the set of real numbers, and the model of the set of areas, which is impossible. In our case, *f* is a many-one function from real numbers out of the intervals (e.g. [m, n], [p, q]) to marked areas, describing a homomorphism between the two models (cf. Suppes, P. [1988]).

It is time to introduce here the concept of *resolution power*, meaning the density of local signs. The resolution power and the interval measure will determine the acuity of constitution. The extreme case would be when reducing the interval to a unique value, the relation of similarity becoming the relation of identity. It is clear now, why Carnap pendulates between these two relations. On the other hand, one can conceive the infinite resolution power, a case in which the constitutional reality would be identical with the external world. It is to mention that, even in the psychological reality there is no infinite resolution power<sup>4</sup>.

We have clarified, so far, the state of affairs on the abstract level. Furthermore, due to the newly introduced concepts, the preliminary level is deciphered. Why, is the temporal order preliminary here? For the Rs-order of the elementary experiences is not exhaustive, to say, there are some elementary experiences not entering the similarity circles (§87). These elementary experiences do not contain as quasiconstituent parts spots belonging to any maximal colour spot class. Their spots have Cartesian coordinates whose values are outside the range of the stipulated intervals.

Ergo,  $c \text{ Rs } y \Leftrightarrow A \text{ ag } B \Leftrightarrow (a1 \notin [m, n] \text{ or } a2 \notin [m, n], \text{ or } b1 \notin [p, q] \text{ or } b2 \notin [p, q])$ , where 'ag' is the agreement relation.

Akin to the temporal gaps, there are, also, qualitative and spatial ones, on this level. Therefore, the visual field cannot be meant as a surface (which has no gaps), but as a bidimensionality only. The qualitative gaps are caused by the spots outside the maximal spots classes.

On the physical level, the three interrupted orders are completed (*vervollständigt*) through the regularities of the physical processes (§87). How are things going on these two levels, physical and that of physics, we will evince in the second part of these inquiries. In this section there has been indicated a possible way of formalizing the quasianalysis, based on the formal concept of *interval*.

### 7. PICTORIAL EXPLANATION (may be omitted)

In this section we compare the constitutional procedure with the process of a film projection. We assume the spotlight is not a proper technical instrument, but a didactic fictitious apparatus, and the recording tape is divided into photogrammes like a photofilm. The p-plane corresponds to the external world, which is to be reconstructed by a projection. The four

<sup>4</sup> The difference between the ideal observers explained by Moulines through the temporal density of the sections of the stream, is explicated here on the basis of degree differences between resolution powers.

levels of the c plane equate to the film, to the screen of the spotlight, to the projected image assumed to be holographic and, respectively, to the theories of the image. The p stream of experience p is analogous with the sequence of the events that are to be shot and thereafter reproduced by projection. The c stream c replicates the succession of the photogrammes (i.e. c elementary experiences c) of the recording tape. The tape represents the abstract methodological level (the technical details and the discrepancies are, in this case, irrelevant). A photogramme is structured as an elementary experience through local signs in quality spots, and being bidimensional, every point of it is denoted by a Cartesian product. Like the quality spot (lacking in any determination), its pictorial correspondent does not have qualitative determinations, because these appear on the screen of the apparatus and, then in the hologramme. The role of the preliminary level is played here by the screen, bidimensional in its turn. The places are the points on the screen which are sets of Cartesian products of local signs. The identically (Cartesian) coordinated points on the tape give a unique point on the screen. The identically coordinated spots determine a quality on the screen. The complete level corresponds to the holographic image, that is three dimensional. The projection is possible by means of light, which realizes the passing from one level to another, as the logical mathematical rules do it for Carnap's system. The pictorial equivalents of spot density is the proper resolution power of the tape-photogrammes, its maximal value being limited by the photon dimension.

### 8. CONCLUSION

In this first part of these inquiries we have deepened the abstract and the preliminary state of affairs. It has been shown that the quasianalysis is a *mediatory theory* (it makes the passing from the abstract to the preliminary level) and a *constructive* one (it constructs the second level). The noteworthy achievement of the AUFBAU is the attempt to constitute the whole world upon an abstract basis that can be reduced to numerical representations. Solely, Carnap did not proceed, in his work, the quasianalysis till the end of it. This has been realized in the present paper. We have displayed how to look at the system as at a *representational tree*. The actuality of the carnapien theory is attested by the fact that this representational program can govern both the kantian transcendental subject and a computing machine<sup>5</sup>. Consequently, Carnap's system is a model of world construction through a general theory of representation. It can be reconsidered and improved

<sup>5</sup> "Intellectual Autobiography", in: Schilpp, P.A. (ed.), *The Philosophy of Rudolf Carnap*, La Salle, 1963, p. 18.

with the help of neurophilosophy, physics, mathematics and so on. Within the following second part of these inquiries we shall scrutinize the other two levels of the theory: the physical one and that of physics. The leading questions are yielded by the meaning of that completeness based on the regularities of the physical processes, and out of the sense of the reconstituted world (dis)continuity. As above, we will explicitate the problems insufficiently discussed by Carnap, and include them at the right place in the representational tree.

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