Colour Similarity in Carnap's AUFBAU

Gabriel Vacariu, Iulian-D Toader

1. Introduction

The relation of similarity plays a central role in Carnap's Konsistnrespecte theorie. We intend in the following paper to take a closer look at this matter. First there is a difference to be mentioned between the proper analysis and the quasianalysis, and the levels they are functional at. Second we emphasize the different behavior of similarity circles used in the two methods, and reach the crucial issue of the stipulated similarity intervals. En fin it is the problem of colour solid tridimensionality that we consider, and our systematic distinction between nuances and sub-nuances.

2. Proper Analysis and Quasianalysis as They Appear in the Two Examples from §70 and §71

2.1. What must first be said about objects and elementary experiences, i.e. entities which are subjected to these two methods

Both proper analysis and quasianalysis have in common, on one side, the fact that they are not starting from our knowledge of the entities, but from an already given list which pairs the entities. On the other side, they share the same formal structure. The first significant distinction here is the one between two kinds of entities entering these procedures. The proper analysis deals trivially with analysable objects, §70:

In the case of proper analysis, we are concerned neither with points that have no properties, nor with unanalysable units, but with objects which have several constituents (or characteristics).

In Carnap's §70 example, he starts from a number of things coloured in various ways with a number of colours. We do not know about these things what colours each of them has. “All we have is a pair-list, i.e., we know only the extension of the relation of colour kinship [...]”

The quasianalysis, on the contrary, regards objects that cannot be directly analyzable on the account of their indecomposability, (e.g., elementary experiences conceived in the framework of the Gestalt-psychology).

Carnap chooses in the §71 example, as analysable units, the “compound” chords. Chords are compound out of tones. Either in this case we do not know what tones enter each of the chords. At the basis of the pair-list is now the relation of kinship between tones. The relation of kinship appearing in both examples holds between entities proper- or quasianalysed. It is au fond a relation of identity between their proper- or quasiconstitu-

---

1 We are profoundly indebted to Professor Ilie Pârvu who awoke our interest in Carnap's philosophy, and supported us all along.

2 University of Bucharest, Faculty of Philosophy (e-mail: vacariu@fil.unibuc.ro).
The same holds also in the case of chords.

2.2. On the essential distinction between two levels of considering the entities: the physical and the phenomenal, and its relation with proper- and quasianalysis

An essential distinction concerns the way we look at the above mentioned entities. The proper analysis deals with physical, while the quasianalysis treats with phenomenal conceived entities.

As a phenomenon, i.e., as it is given in sensation (in contrast to the viewpoint of physics and acoustics), a chord is a unitary totality (einhheitliche Totalität) which is not compound of constituents.

From the physical point of view, entities (objects, chords) are compound (out of colours and tones, respectively). But this from the physical viewpoint only. As such, only at this level it is possible a proper analysis, i.e., a division of entities in their properies. For example, a chord is formed 'acoustically speaking' out of the concomitant c, e, g tones. From the phenomenal point of view, entities are not taken to be composed of constituents, but as wholes (in the Gestalt manner).

In hearing the chord, we detect — provided that we have already heard a sufficient number of other chords — three constituents, not in the sense of parts, but in the sense of three different directions in which we can proceed from it to other chords (i.e., to entire chord classes which stand to one another in the relation of tone kinship).

In other words, a chord cannot be taken isolated. In order to distinguish its three directions, one must associate this chord with the others previously heard. The chord c — g is not determined by simultaneously hearing the three tones (as in the case of oper analysis), but only through relating it to other chords which contains at least one of a, e, g tones. The chords are compared, as we already mentioned, through the relation of tone kinship. On this basis the chord classes, i.e. the similarity circles, are formed.

2.3. The different behaviour of similarity circles; what they really are and significantly presuppose

To sum up, all we have as a method is a) the proper analysis (based on the relation of kinship between entities, i.e. objects, chords) which is, in fact, a relation of identity between their properties, and b) the quasianalysis (also based on the kinship relation between entities, i.e. an identity relation between their properties). Carnap speaks about “a second type of quasianalysis”, too. This second type, unlike the first one, is based on the relation of similarity (§72). What one must keep in sight is that similarity circles do not behave the same in the two cases. In proper analysis and in the first type of quasianalysis similarity circles identify themselves as quality classes.
The task of analysis is attained once we succeed in determining the “colour classes”. Let us call the class of all things which have a certain colour in common a “colour class” (e.g., the class of the red [completely red or also red] things, of blue things, etc.). §70

In the case of quasianalysis based on the Part similarity relation, similarity circles are the bridge to the quality classes.

We call two things colour similar if, among other colours, they each have a colour which is similar to that of the other (i.e., which, on the colour solid, has a distance from the other which is smaller than a certain arbitrarily chosen magnitude). […] It is impossible in this case to determine directly the colour classes (i.e., the classes of all and only those things which, among other colours, bear a certain colour). §72

So, we will determine first and easier a different type of class, the colour similarity circles. If that is the case, why cannot we determine directly the colour classes? Because here we do not deal with the same colour assigned to all entities. Rather, it is a matter of variegating concerning brightness, saturation and hue of that colour. The similarity circle is a too large class for denoting univocally a colour. It is necessary to implement a further condition in order to set aside the above-mentioned variegating and to get a single quasiquality. Let us take an example. We suppose there are two chords. One can speak about the first type of quasianalysis when one of the tones is the same (it has the same pitch and loudness) in both chords. The common tone is the wanted quality. Now, the second type of quasianalysis handle with only similar (that is, not identical) tones, i.e., with differences in pitch and loudness. On the one side, two chords a, b, c and d, e are related through the tone c (by Part identity). On the other side, the two chords a, b, c' and d, e are related by Part similarity between c' and c" (c' and c" are two variations in pitch and loudness of the tone c). On account of this difference one cannot determine simply the quality class c. Nonetheless, the similarity circle is allowed to contain chords where c has the maximum and respectively, the minimum value in pitch or/and loudness, if we take the similarity interval (see section 2. below) to be the largest possible one. Actually, in the Konstitutionsystem we do not deal with such large intervals, but with finer ones because only these can account for c tone alike qualities. That’s why the interval needs to be restrained and formerly stipulated (festgesetzt). ¹

¹ This restraint implies avoiding the possibility of irregularity in the phenomenal world (cf. Moulines (1991), p. 283).
3. THE CRUCIAL ROLE PLAYED BY COLOUR SPHERES AND THE WAY THEY LEAD TO THE KEY-CONCEPT OF CARNAP'S SYSTEM: THE STIPULATED INTERVAL

3.1 Where it becomes clear the meaning of the colour solid tridimensionality and its connection with the similarity interval

In §72 example, Carnap defines the colour solid which is tridimensional; these three dimensions are not to be understood as spatial, but related to brightness, saturation and hue. The colour solid represents exactly the possibility of one particular colour to variegate in these three characteristics: hue (chromatics variations of colour), saturation (intensity-changes of it; Carnap understands by saturation the white content of a colour), brightness (i.e., luminosity variations, black content in Carnap's words). The set of colour solids forms the colour matrix. For example, the green can largely variegate in hue between the closest to yellow nuance of green and the closest to blue nuance of green. In this interval there are only similar green nuances. In saturation, the green varies as the colour resolution changes (that is, the minimum value corresponds to the least intense colour). Finally, brightness takes the green between the closest to white and the closest to black green nuances.

Further, Carnap defines colour spheres having an essential role in the constitutional system economy.

The largest possible parts of the colour solid, which contain nothing but colours that are similar to one another, are spheres which partially overlap each other, and whose diameter is the arbitrarily fixed a maximal distance of similarity (which may be different in different parts of the colour solid).

This stipulation takes places on each of those three dimensions. Two colours are similar if they are similar in chromatics, resolution and luminosity, i.e., whether they belong to the same similarity interval. They do not belong to it because they are similar, but conversely they are similar because they belong to one and the same interval. Now, it naturally arises the question how can I put together two colours in to the same interval, knowing nothing about them? One possible answer is that they could be included in to the interval through the comparison of the Cartesian coordinates of the local signs which correspond to the two colours (cf. Toader/Vacariu, 1997).¹

3.2 Nuances, sub-nuances and the perils of the infinite regression

Inside the same colour solid (that corresponds to a certain colour), colour spheres differ among them by nuance (see fig. 1) and diameter (S₁+S₂+S₃+S₄+S₅).

¹ We regard now this answer as a not so carnapian one, and try to offer here a solution closer to AUFBAU.
Let us take one of these sphere $S_i$ in order to see how it is obtained. A colour sphere is formed by colours, i.e., out of similar sub-nuances (see below), those that belong to the similarity interval that defines the diameter of our sphere. A colour similarity circle is made out of things, which have one of the colours of a certain colour sphere (§72), that is one of the sub-nuances of our sphere. Each nuance can vary on the three dimensions, in this way the sub-nuances are coming out. The sub-nuances of a certain colour nuance represents the smallest variations of that nuance in saturation, brightness, hue. Therefore, colour similarity circles are actually sub-nuances similarity circles. The nuance of a colour sphere is obtained through an essential overlapping of sub-nuance similarity circles.

For an example, let us consider a blue colour solid, and therein a sphere of a nuance of blue, $S_1$. This nuance is determined by essentially overlapping the sub-nuance similarity circles, let them be three: $S_1'$, $S_1''$, $S_1'''$ (fig. 2).

There is still unclear how it is possible for two things in the same sub-nuance similarity circle to be part similar. Two things are in the same $S_1'$-similarity circle if both of them have colours belonging to the interval that determines the sphere $S_1$, and more, belonging to the interval which determines the similarity of the different smallest variations of $S_1'$.
Now, the colour classes are to the individual places of the colour solid, what the colour similarity circles are to the colour spheres. Since the individual places of the colour solid are the largest parts of the colour solid, which remain always undivided in the mutual overlapping of the colour spheres, we can determine the colour classes correspondingly as the largest subclasses of the colour similarity circles which remain undivided through the mutual overlapping of these circles. (§72)

Fig. 3 The marked areas are locations (Einzellstellen) of the colour solid, viz., that what remains outside any intersection from the colour classes.

3.3 Essential and accidental overlapping. What is that all about?

As seen above, the overlapping of the sub-nuance similarity circles is an essential one. Carnap defines in §82 another type of overlapping: the accidental. To clarify this one let us take two sub-nuance similarity circles $b_1$ and $b_2$ (cf. §81), $b_1 = \{b_{11}, b_{12}, b_{13}\}$, and $b_2 = \{b_{21}, b_{22}, b_{23}\}$, where $b_{ii}$ are the smallest variations of $b_i$ and $b_2$, respectively. Let $q = \{x | x \in b_1, x \in b_2, b_{11} \subseteq x\}$ be a colour class, where $x$ is an elementary experience. Let $y$ be another elementary experiences, $y \not\in q$; $z$ is an elementary experience that does not belong to $q$, but includes $b_{12}$. The $q$ — class is formed by two types of elementary experiences: those including $b_{11}$, and those including both $b_{11}$ and $b_{21}$ (or another variation of $b_{21}$). Subsequently, $q$ is sectioned through the overlapping of the two similarity circles.

In the case of the essential overlapping $q$ — class must be wholly comprised in the common intersection of the two similarity circles. In the §81 example, Carnap says:

---

1 This stops the infinite regression, possible on account of the similarity relation.
We are here concerned with an accidental overlap between a and b (in our case b1 and b2 — our note, I-D.T./G.V.): it cannot be an essential overlap in this case, since a and b belong to different colour solids and furthermore to different colour ranges within the colour solids.

So, an essential overlapping can occur only when b1 and b2 belong to the same colour solid, and more, therein only when they belong to the same colour sphere.

**4. AS A MATTER OF CONCLUSION**

In this paper, we tried to emphasise as the keynote of Carnap's system the concept of similarity interval, an interval expressed by numerical values, both on the three qualitative dimensions and on the two spatial dimensions. One must notice that, from a holistic point of view, between the elementary experiences there could exist two kinds of similarity:
(a) at temporally minimal intervals (we call this the 'infinitesimal' level of similarity),
(b) at temporally larger intervals.

It emerges now as obvious the great importance of the local sign (i.e., of Cartesian coordinates) in the economy of the system: we believe Carnap introduces the two spatial dimensions of a colour in order to restrain the similarity to the case (a). Only in this case the similarity of two elementary experiences involves also a similar spatial position of colours.

**5. REFERENCES**